

What are K-5 Students Learning and When?

All-City Tutors 10/19/19

Key Content: K – 5

Counting: K – 1st

Whole Numbers

Add & Subtract: K – 4th

Standard Algorithms – 4th grade

Multiply & Divide: 3rd – 6th

Multiplication Standard Algorithm – 5th grade

Division Standard Algorithm – 6th Grade

Round & Estimate: 3rd – 4th

Fractions

Words & Equal Pieces: 1st – 2nd

Half & Fourth – 1st

Third – 2nd

Unit Fractions & Comparing: 3rd

Equivalent Fractions: 3rd – 4th

Addition & Subtraction: 4th – 5th

Like denominators (including Mixed Numbers) – 4th

Unlike denominators (including Mixed Numbers) – 5th

Multiplication: 4th – 5th

Whole number × Fraction and Whole × Mixed Number – 4th

Fraction × Fraction (and Mixed Numbers) – 5th

Division: 5th – 6th

Whole number ÷ Unit Fraction and Unit Fraction ÷ Whole number – 5th

Fraction ÷ Fraction – 6th

Decimals

Writing Tenths & Hundredths: 4th

Addition & Subtraction: 5th – 6th

Within thousandths – 5th

Standard Algorithm – 6th

Multiplication & Division: 5th – 6th

Within thousandths (tenth × hundredth) – 5th

Standard Algorithm – 6th

Key Strategies Throughout all Operations & Number Types

Decompose Numbers – separate them into smaller pieces

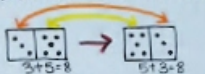
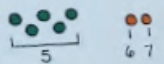

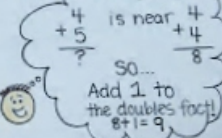
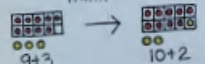
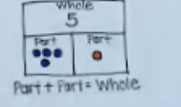
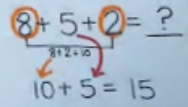
Properties of Operations: Commutative, Associative, Distributive (students do not need to know names)

Connections between Operations

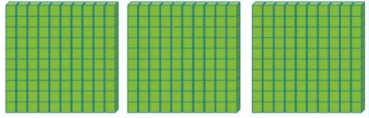
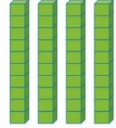

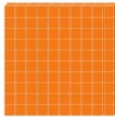
Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Properties, strategies, and modeling.

<p>Zero Facts</p> $12 + 0 = 12$ $0 + 12 = 12$ <p>Any number + 0 = Same number No change!</p>	<p>Turn-Around Facts</p>  <p>Flip the addends... The sum DOESN'T change! $3 + 5 = 5 + 3$</p>	<p>Counting On</p>  <p>* Put the bigger number in your head then count up! +1, +2, +3 facts</p>
<p>Doubles</p>  $4 + 4 = 8$ <p>the same number is added together!</p>	<p>My Addition Strategies Mat</p>	<p>Near Doubles</p>  <p>So... Add 1 to the doubles fact! $8 + 1 = 9$</p>
<p>Make A Ten</p> $9 + 3 = ?$ <p>Think!</p>  $9 + 3 = 10 + 2$	<p>Part-Part-Whole</p>  <p>Part + Part = Whole</p>	<p>Combine Numbers</p>  $8 + 5 + 2 = ?$ $10 + 5 = 15$

Core Lesson What is $341 + 156$?

Hundreds	Tens	Ones
		
		

I can subtract two-digit numbers.

$45 - 17$

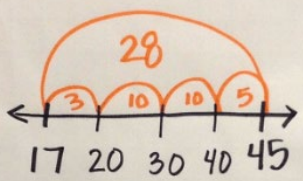
Think Addition

$$17 + 3 = 20$$

$$20 + 20 = 40$$

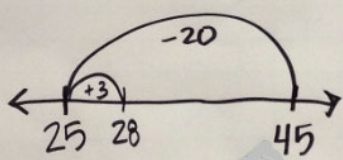
$$40 + 5 = 45$$

28



Compensation

$$45 - 20 = 25$$

$$25 + 3 = 28$$


Decompose a Ten

45 \rightarrow 40 + 5 \rightarrow 30 + 15
 \rightarrow -10 - 7
 \leftarrow 20 + 8

shortcut

$$\begin{array}{r} 3 \ 15 \\ 45 \\ -17 \\ \hline 28 \end{array}$$

7 x 16 =

$7 \times (10 + 6) =$

70 + 42 = 112

143 x 27

	100	40	3
20	2,000	800	60
7	700	280	21

3,180 ÷ 15 = 213

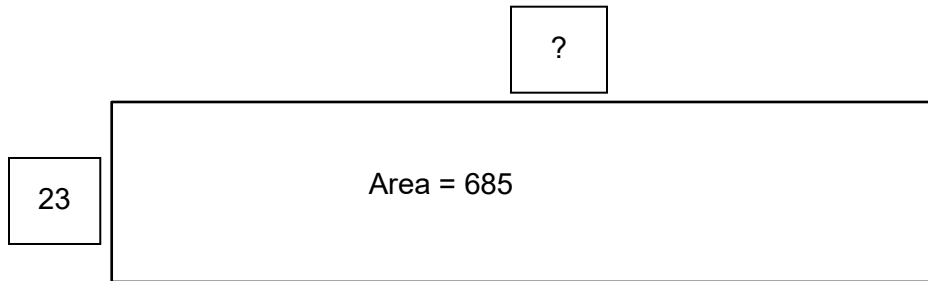
	200	10	3
15	3,000	150	30

$$\begin{array}{r} 15 \overline{) 3,180} \\ \underline{-3,000} \quad 200 \\ 180 \quad 10 \\ \underline{-150} \quad 3 \\ 30 \quad 3 \\ \underline{-30} \\ 0 \quad 213 \end{array}$$

Partial Quotients

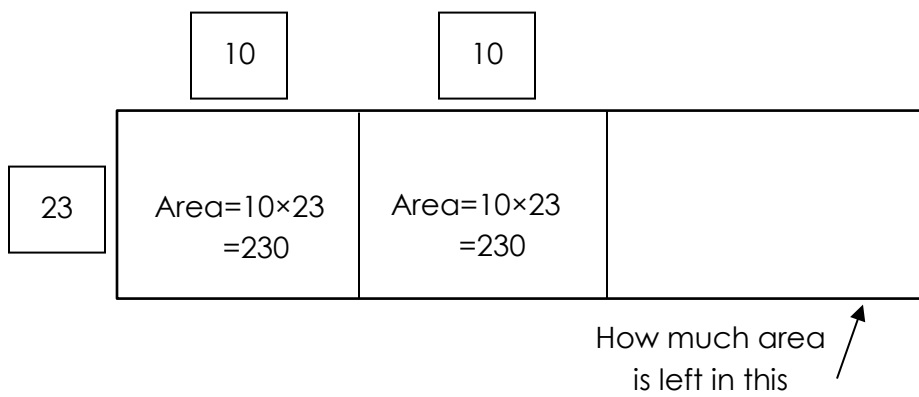
2. Find $685 \div 23$.

a) From the equation you know the total area is 685 and one dimension of the rectangle is 23. How?



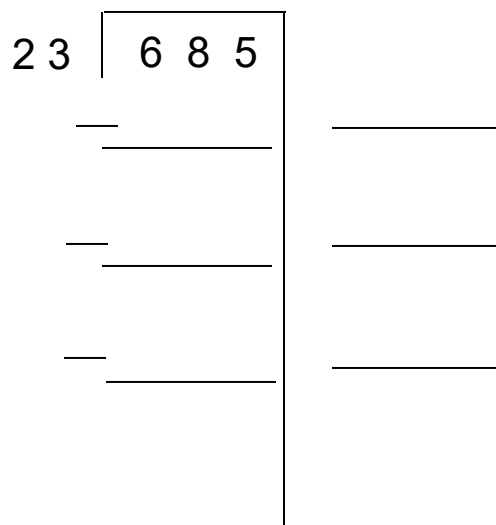
Now figure out how long the other side is by dividing it into reasonable chunks. As students gain number sense, they will make more efficient estimates.

Here's a start on figuring out the second side length.



Use your estimating skills to figure out how much longer the top side of the rectangle is. You might need to use more than one estimate and add another box.

b) Here's another way to record partial quotients: (sometimes called *The Big 7*)



Three Reads Notetaker



1st Read. What is the problem about?



2nd Read. What is the question?



3rd Read. What is the important Information?

Math Constructive Conversation Skills Poster

Clarify Problem and Ideas for Solving It



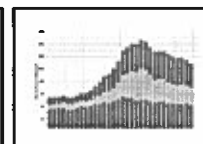
Prompt starters:

What are we trying to do?
 What is the problem asking?
 How does the problem begin?
 What happens in the problem?
 What do we need to know?
 How can we break this down?
 What type of problem is this?
 What patterns do we notice?
 What's a possible plan for solving it?
 What is your estimate for the answer?
 Why are you doing that?
 Where did that number come from?

Response starters:

In order to ____, we need to ...
 In other words,
 More specifically, it is ... because...
 Let's see, it is similar to the problem about ... that we did because...
 It is important to ____ because
 Let's stay focused on
 Let's get back to the idea of...
 In future problems like this one we need to remember to...

Generate & Try Multiple Methods & Representations



$$\int_0^{\infty} \sum_{i=0}^{\infty} \frac{A_i(x)}{2\pi}$$

Prompt starters:

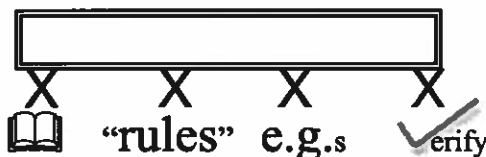
How else can we show this?
 How can we draw or graph this?
 What symbols can we use?
 How can we explain this to others?
 How can we write what we are thinking/doing?
 How can we translate this into symbols?
 Let's back up and try a different way.
 Which method is most useful? Why?

Response starters:

Maybe we can use...
 Another way to show this is...
 In math symbols we could write...
 We can draw it like this because it says...
 Let's try to... and see what happens.

Build Math Ideas & Understandings

Explain & Support Reasoning



Prompt starters:

Can you explain why you...?
 What does that mean?
 What are you doing?
 What math rule are you using?
 Can you give an example?
 How does the sample problem help us?
 What are examples of this problem from real life?
 Can you clarify where you...?
 How did you get this answer?

Response starters:

If we ____, then we need to ____ because...
 A key mathematical principle is making sure that you...
 In real life this is similar to when you want to...
 An example from my life is
 One case that illustrates this is...
 In math, we always need to...
 Let me show you what I mean.
 We can't do that because it...

Negotiate Ideas



Prompt starters:

How can we add to the idea of...
 What do you think about this strategy for solving it?
 What else could support this idea?
 Do you agree?
 What contradicts this? What are other points of view?
 What did we learn from doing this problem? How will it help in life?
 Let's create a similar problem.

Response starters:

That reminds me of...
 I want to add on to your idea of...
 That idea connects to...
 I see it a different way,
 On the other hand, ...
 That makes me think of...
 We can agree that...

Learning to Question and Questioning to Learn in Mathematics

Persevering

- How would you describe the problem in your own words?
- What facts do you have? What do you know that is not stated in the problem?
- How did you tackle similar problems?
- Could you try it with simpler numbers? Fewer numbers? With a number line?
- What about putting things in order?
- Would it help to create a diagram? Make a table? Draw a picture?
- Can you guess and check?
- Have you compared your work with anyone else?

Problem Solving

- What information do you have? What do you need to find out?
- What strategies are you going to use?
- Will you do it mentally? With pencil and paper? Using a number line, table, diagram or picture?
- Will a calculator help?
- What tools will you need?
- What do you think the answer or result will be?

Reasoning & Justifying

- Can you tell me why that is true?
- How did you reach your conclusion?
- How does your answer connect to the question? Does it make sense?
- Can you make a model to show that?

Communication & Collaboration

- What do you think about what _____ said?
- Do you agree? Why/why not?
- Does anyone have the same answer but a different way to explain it?
- Do you understand what _____ is saying? Can you explain what _____ is saying?
- Can you convince the rest of us that your answer makes sense?

Reflection and Learning from Errors

- How did you get your answer? Can you explain why your method works?
- Does your answer seem reasonable? Why or why not?
- What if you had started with...rather than...? What if you could only use...?
- What have you learned or found out today?
- What new words did you use today? How did you use them?
- What did you learn from that mistake/error? How did you know it was an error?
- Can you explain where your thinking changed?
- What are the key points or big ideas in this lesson?

Taking Responsibility

- How are your notes/notebook organized?
- How you identify what is important to remember? (colors, shapes, etc)
- How does what you learned in class today connect to what you already knew?
- How do you identify what you have questions about?
- How do you keep track of new vocabulary?

Handout - Seven Common Growth Mindset Scenarios and Responses

As a mentor, you will encounter multiple situations where you can encourage and reinforce a growth mindset. But even if you understand the concepts of growth mindset, it may not always be clear what to say when confronted with a student who is struggling to persevere or who is shying away from challenges. This tip sheet illustrates some of the messages you can deliver about mindset in response to common situations you may face as a mentor.

<p>Situation 1: Faced with a new learning challenge</p> <p><i>Underlying principles:</i></p> <ul style="list-style-type: none"> • Challenges are exciting, not just overwhelming. • Effort is important: you'll get out of this what you put into it. • Having a strategy is vital. <ul style="list-style-type: none"> ○ Divide the learning into pieces that can be taken as chunks and defining them ○ Set up opportunities for there to be small wins that lead to the completion of the larger learning goal • It's OK to ask for help. A little struggle is a sign we are stretching and leaving our comfort zone. But after a while, it's OK to get help or hear new strategies. 	<p>Some potential responses:</p> <ul style="list-style-type: none"> • Let's identify a target for today that will get you closer to completing the learning challenge? After you complete the day's target, what might tomorrow's target be? • This is a great challenge! Your brain is going to get stronger as you work through the challenge. • Let's take one step at a time that way we can see where we might need to focus more attention and time. • This looks like pretty demanding stuff. What would a focused first try look like? • I am here to help you learn how to ... • Let's come up with a strategy. • Describe this challenge in your own words. Share anything that might be really confusing. • This may be difficult now, but might be a lesson you remember for the rest of your life. • I have seen you learn challenging things in the past. For example, last _____ [week/month] I saw you... • This is challenging! What do you think are some strategies you could try?
<p>Situation 2: Expressing high expectations</p> <p><i>Underlying principles:</i></p> <ul style="list-style-type: none"> • The research is clear, setting high expectations tells kids the adults they care about believe in them • Unrealistically high expectations without support; however, are a different matter 	<p>Some potential responses:</p> <ul style="list-style-type: none"> • Let's think through this to determine what you know and where you might need support. • Let's discuss some strategies for tackling this. • What do you already know about this? • When you learn this/do this/ succeed at this, you can be proud because it isn't easy.

	<ul style="list-style-type: none"> • This looks like one of those opportunities to stretch/to reach higher. • This is a challenge that could produce some great mistakes that will really help you learn.
<p>Situation 3: Succeeding easily without effort</p> <p><i>Underlying principle:</i></p> <ul style="list-style-type: none"> • Having it be too easy is counterproductive • Acknowledging the lack of challenge and determining the appropriate level of challenge is important 	<p>Some potential responses:</p> <ul style="list-style-type: none"> • You finished that quickly. Let's find something a little more challenging. • That seems a little easy for you. How can you make it a stretch enough to build your brain? • I am sorry this was not challenging for you. Your skills didn't seem taxed. Is that true? • What can you do to make this [more meaningful, challenging, exciting]? • How can you add another level to this to challenge you even more? • Do you already know how to do this? Let's come up with something more challenging if you already know how to do what was presented.
<p>Situation 4: Slow progress despite strong effort</p> <p><i>Underlying principle:</i></p> <ul style="list-style-type: none"> • Effort is the key to success • Identify supports to help foster a sense of success and accomplishment • Analyze the strategies being used and see if they can be improved (see below) 	<p>Some potential responses:</p> <ul style="list-style-type: none"> • I see that you tried that five times. I admire your persistence. It will pay off. • Let's review all of your attempts to determine the best course of action. • Remind yourself that you just can't do it "YET." Let's think through some next steps to take. • Let's walk through the problem/assignment/issue/task, perhaps you need a little more information or guidance to get to the next step. • If it were easy, you wouldn't be learning enough. • What progress did you make? What was different? • I expect you to make mistakes. Mistakes are the signals of opportunities for learning - what did you notice in the mistakes you made? Is there anything in the mistakes that will identify where you might need additional guidance or support? • Does it make sense to stop now and come back to it later? • Let's talk about how you've been approaching the problem/assignment/

	<p>issue/task. Maybe one of your strategies could be improved.</p>
<p>Situation 5: Offering help with strategies when struggling</p> <p><i>Underlying principle:</i></p> <ul style="list-style-type: none"> When challenge because difficult and your mentee wants to give up, support him or her by identifying strategies that will support persistence and resilience 	<p>Some potential responses:</p> <ul style="list-style-type: none"> Okay, let's think about how to approach this differently? Would you like to try _____ [different strategy]? Let's try it together. Let's do it a few more times to get the synapses strong - get that learning into long-term memory. What was difficult? Let's focus on the difficulties to see if we can figure it out. Who else can you ask for help? Let's put a plan together for the next _____ [days, weeks]. Let's go through it together and find the mistakes. What was your approach? Where do you think you might be struggling the most? Let's de-stress, so your brain can relax and process better [square-breathing, changing the physical location in which the work was being done, etc.] Does it make sense to stop now and come back to it later?
<p>Situation 6: During progress</p> <p><i>Underlying principle:</i></p> <ul style="list-style-type: none"> As your mentee begins to make progress toward a goal or an important task, it's important to praise the process in order to build persistence 	<p>Some potential responses:</p> <ul style="list-style-type: none"> Show me how you arrived at your present conclusion? Starting to come along nicely - your strategy is working! It seems like the problem/task/concept is at a right level; you've been working on it for a while. Good job! I see you are using your notes. What other strategies have you used or could you use to continue to make progress? I can see a difference in now compared to _____ from last week/yesterday. What has changed? Talk me through what happened.
<p>Situation 7: Succeeding with strong effort</p> <p><i>Underlying principle:</i></p> <ul style="list-style-type: none"> It is important to acknowledge the effort once a new challenge is overcome and complete 	<p>Some potential responses:</p> <ul style="list-style-type: none"> What was it like for you when you started work on _____? Look how different it is for you to do that now. Did all that hard work pay off?

- When mentees understand that they have strategies in their toolbox for tackling big challenges, they will be able to use specific tools for specific challenges

- What do you think contributed to your success in _____?
- I saw you use a variety of techniques; way to go!
- This had that one brilliant mistake. Let's talk about what you learned from that mistake.
- Did you compromise on anything to get this done?
- The next time you have a challenge this big, what can you use from this experience.
- Congratulations for trying again and again to get this done.
- How would you compare this to other accomplishments?

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